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Artificial Intelligence

Homework 3

1 (10 points) There is a propositional knowledge base with only unary and binary clauses. How will you check if a sentence of form $A \rightarrow B$ is entailed by this knowledge base or not by using resolution rule minimum number of times? A and B are both propositions.
evaluate $\neg A \vee B$

2 (40 points) A first-order knowledge base contains the following sentences.
(i) $\forall x(P(x) \rightarrow Q(x))$
(ii) $\forall y(Q(y) \rightarrow M(y))$
(iii) $\forall y((Q(y) \vee M(y)) \rightarrow R(y))$
(iv) $\exists z P(z)$
Answer if this knowledge base entails the following sentences.
(a) $P(A)$
No, it contradicts $\exists z P(z)$
(b) $P(B) \rightarrow Q(B)$
Yes, the knowledge base entails this sentence. $\forall x(P(x) \rightarrow Q(x))$
(c) $P(B) \rightarrow M(B)$
Due to...
$\forall x(P(x) \rightarrow Q(x))$
$\forall y(Q(y) \rightarrow M(y))$
No. The domain is different for P(B) and M(B)...

(d) $\forall x(P(x) \rightarrow R(x))$
$\forall x(P(x) \rightarrow Q(x))$
$\forall y(Q(y) \rightarrow M(y))$
$\forall y((Q(y) \vee M(y)) \rightarrow R(y))$
No, the knowledge base does not entail this sentence.

3 (30 points) A knowledge base contains the following 5 sentences.
(i) $\forall x(L(x) \rightarrow D(x))$
(ii) $\forall x((D(x) \wedge A(x)) \rightarrow MC(x))$
(iii) $\exists x(L(x) \wedge \neg C(x))$
(iv) $\forall x((A(x) \wedge \neg F(x)) \rightarrow C(x))$
(v) $\forall x(L(x) \rightarrow A(x))$
Can you derive the following sentence from this knowledge base by using resolution-refutation technique? If so, show the proof in first-order logic.
Mention the inference rules used for various steps.
Must find contradiction using NOT goal with existing knowledge base, which therefore proves the original goal state.

∃x(D(x) ∧ A(x) ∧ F(x))
First get NOT goal...
P(X) =(D(x) ∧ A(x) ∧ F(x))
¬∃xP(x)
∀x¬P(x)
G: ∀x(¬D(x) v ¬A(x) v ¬F(x))

∀x((A(x) ∧ ¬F(x)) → C(x))
∀x(¬(A(x) ∧ ¬F(x)) v C(x))
∀x(¬A(x) v F(x) v C(x))
∀x(A(x) v F(x) v ¬(D(x) ∧ A(x)))
∀x(A(x) v F(x) v ¬D(x) v ¬A(x))
∀x(A(x) v F(x) v ¬D(x))
Therefor there is a contradiction and KB ∧ ¬G is unsatisfiable, proving G.

4 (20 points) Does the following knowledge base entail ∃xR(x)? Use resolution in your proof and mention names of other inference rules you use.

  (i) ∀x(P(x) → (Q(x) ∨ R(x)))
  (j) ∀y¬Q(y)
  (k) ∃zP(z)

Yes
∀y¬Q(y) => ¬∃yQ(y)...∀x(P(x) → (R(x)) and ∃zP(z)